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# The Vehicle Routing Problem: Applications and Research Perspectives

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**Abstract**

**This is an expository article about the Vehicle Routing Problem, which is an important problem in the area of supply chain management. The purpose of this article is to draw the attention of potential researchers towards this problem. Even though the problem is heavily researched, there is plenty of scope for more research on this problem because many new versions of the problem keep emerging. After introducing the problem, the article provides a mathematical formulation of the problem, an illustrative example, and a brief literature survey. Then a number of different versions of the problem which are of practical interest have been discussed. Finally, the article ends with a short discussion on the scope of future research on this problem.**

*Keywords:* Supply Chain, Vehicle Routing, Integer Programming

## 1. Introduction

In supply chain management, a common scenario is one in which the goods have to be transported from a central location, e.g. a warehouse, plant, or a depot, to a number of demand points, e.g. customer locations, that are geographically spread around the depot. The quantities to be delivered are typically a small fraction of the vehicle capacity. Therefore, a vehicle starting from the depot makes deliveries to multiple customer locations, and then returns back to the depot. As the vehicle capacity is limited, multiple vehicles will be required to make deliveries to all customers. Each customer has a specific demand quantity, which must be delivered by a single vehicle. In other words, splitting the demand of a customer and delivering it in multiple visits is not permitted. Thus, each customer location is visited by exactly one vehicle.

It is easy to see the decision problem faced by the management in such a scenario. Assuming that all the vehicle have equal capacity $C$, and the demand of customer $i$ is $w_i$, it must be ensured that the total demand of the customers visited by any vehicle does not exceed the capacity of the vehicle. The decision maker must decide as to which customers should be assigned to which vehicle, and in what sequence should each vehicle visit these customers, so as to minimize the total cost of making all deliveries. There may be several cost components in such a scenario, but the dominant one is the operating cost of the vehicle, which is typically proportional to the distance travelled by the vehicle. The depot node is indexed by 0, and we denote with $c_{ij}$ the cost of travel between nodes $i$ and $j$.

The importance of this problem lies in the fact that the distribution costs constitute a significant component of the cost structure of companies. If this cost is not efficiently optimized, it may significantly affect the profits as well as the competitiveness of the company. Moreover, the underlying mathematical problem involved in this operation is by no means trivial to solve, and any casual attempt to find a workable solution may often produce a very bad solution.

The decision problem described above, with all its myriad versions and variations which we shall introduce in the rest of this paper, is called the Vehicle Routing Problem (VRP). It can be modeled as a binary integer program. Although there are

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several alternative ways of formulating the mathematical problem, here we describe a popular formulation, called the directed flow formulation. Let the binary variable $x_{ij}$ be equal to one, if a vehicle travels from node $i$ to node $j$, and zero otherwise. In addition, we need continuous variables $W_i$ that denote the load on the vehicle immediately before it reaches node $i$. Then, the formulation is:

$$\text{Minimize} \sum_{i=0}^{n} \sum_{\substack{j=0 \\ j \neq i}}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j \neq i} x_{ij} = 1 \quad \forall i = 1, .., n \tag{1}$$

$$\sum_{j \neq i} x_{ji} = 1 \quad \forall i = 1, \ldots, n \tag{2}$$

$$W_i \geq w_i x_{i0} \quad \forall i = 1, \ldots, n \tag{3}$$

$$W_i \geq W_j + w_i x_{ij} - C(1 - x_{ij}) \quad \forall i, j = 1, .., n \tag{4}$$

$$W_i \leq C \quad \forall i = 1, \ldots, n \tag{5}$$

$$x_{ij} = 0, 1 \tag{6}$$

$$W_i \geq 0 \tag{7}$$

The objective function minimizes the total distance travelled by the vehicles. Constraints (1) and (2) ensure that each customer node has exactly one vehicle exiting and entering the node, respectively. Constraint (3) says that if node $i$ is the last one on the route (i.e. $x_{i0} = 1$), then the load on the vehicle before entering node $i$ is not less than the demand of that node. Note that if $x_{i0} = 0$, then the constraint becomes redundant, as it only implies the non-negativity of $W_i$. Constraints (4) say that if the vehicle moves from $i$ to $j$ (i.e. $x_{ij} = 1$), then $W_i$ should be no less than $W_j + w_i$. On the other hand, if $x_{ij} = 0$, again the constraints becomes redundant, as the RHS becomes zero or negative. Constraints (3) and (4) together ensure that at every stage the vehicle has enough load to fulfill the demands of all the remaining nodes to be visited on the route. Constraint (5) simply enforces the vehicle capacity constraint. Constraints (6) require variables $x_{ij}$ to be binary, and (7) enforces non-negativity on $W_i$.

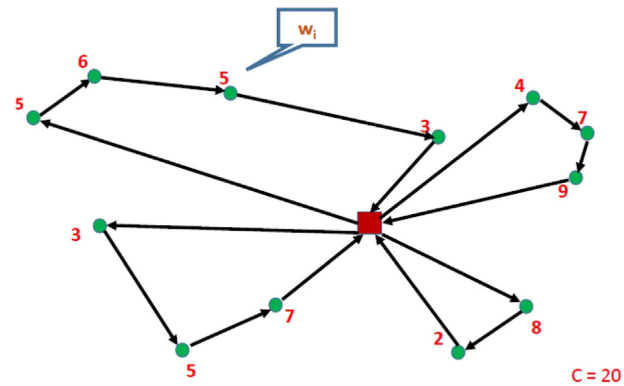


*Fig. 1. A typical vehicle routing problem, and its optimal solution.*

A typical 12-node VRP and its optimal solution is displayed in Fig. 1. The number adjacent to each nodes depicts the demand of that node.

## 2. Literature review

Being a problem of immense applied interest, there exits a vast body of research on this problem. We mention only a few significant contributions here. The problem was first introduced by Dantzig and Ramser (Dantzig & Ramser, 1959), who called it the *truck dispatching problem*. A very famous early paper by Clarke and Wright (Clarke & Wright, 1964) introduced a very simple and effective heuristic approach for solving the problem, which is being used in many commercial software packages even today. Christofides, Mingozzi and Toth (Christofides et al., 1981) proposed an exact algorithm based on spanning tree and shortest path relaxations. Agarwal, Mathur and Salkin (Agarwal et al., 1989) gave the first set partitioning based model solved via the column generation method. An early survey paper on the problem was given by Laporte (1992) which reviewed the exact as well as approximate algorithms on the problem. Interested readers may refer to the recent book by Toth and Vigo (Toth & Vigo, 2014), which surveys in great detail the numerous variants of the problem, and the solution methods proposed by various researchers.

## 3. Relationship with other optimization models

Most readers must already be familiar with the Traveling Salesman Problem (TSP), an extremely well-known optimization model. In this problem, we are given a set of locations/nodes and the distances/travel costs among those locations. Starting from any arbitrary node, it is required to find the least cost sequence in which all the nodes should be visited, and we return back to the starting point. Each node must be visited exactly once on the route. This problem is considered a very hard problem to solve, and a large body of research exists on this problem.

It is easy to see that the TSP is actually a special case of the VRP, which means that under certain conditions the VRP becomes the same as the TSP. Consider a VRP in which the vehicle capacity is sufficiently large, so that all customers can be served by a single vehicle, i.e. $C \geq \sum_i q_i$. Then, clearly, the problem is nothing but a TSP.

It is noteworthy that the VRP, in general, is a much harder problem to solve than the TSP. TSPs with hundreds or even thousands of nodes are easily solved with today's computers within a matter of minutes, if not seconds. However, some VRPs with even less than 100 customers may take hours, if not days, to find the optimal solution.

The reason for this complexity of VRP is that it contains another optimization problem called the Bin Packing Problem. Given a number of objects of various sizes, and bins of a fixed size, the Bin Packing problem is to find a packing of the objects into bins which uses the smallest number of bins, making sure that the total size of the objects packed into any bin does not exceed the capacity of the bin. Each object must be packed into a single bin, and cannot be split into two or more pieces and then packed into multiple bins.

In the VRP, suppose that each truck is hired from the transporter at a fixed daily cost irrespective of the distance the truck is driven. Then, the distance travelled by the vehicles is irrelevant, and the objective becomes to minimize the number of vehicles required. It is easy to see that in this scenario, the problem reduces to a bin packing problem. Both TSP and Bin Packing Problem are difficult combinatorial problems to solve in their own right. That is why the VRP becomes a much harder problem to solve as it combines the features of both of these problems.

## 4. Applications and variants

There are numerous real world applications of the above model. Any time deliveries are made to customers in less than truckload (LTL) quantities from a central location, the above model, or a variant of it emerges. In any city, the oil companies make deliveries of petroleum products such as petrol or diesel to the pumping stations from a central depot generally located at the outskirts of the city. Each pumping station conveys its requirement to the depot each day, which changes from day to day. Accordingly, the depot manager has to plan the routes of the vehicles to fulfill all requirement at minimum possible cost. The underlying problem is the same whenever a wholesaler is making deliveries to the retailers in LTL quantities. Similar applications arise in routing of milk vans, courier deliveries, school buses, and so on.

In some instances, rather than delivering, the vehicle may be collecting the goods which have to be brought back to the depot. Some examples are: collection of raw milk from villages to a processing plant, or collection of garbage from various localities in the city. The underlying mathematical problem in this case remains exactly the same as modeled above.

The model described in Section 1 is called the standard VRP or the Capacitated VRP (CVRP). Numerous variants of this basic problem arise in real life, many of which have been extensively studies in the literature. We briefly describe some of the commonly arising variants next.

### 4.1. Vehicle routing with time/distance constraint

In this variant, the total time/distance of each vehicle route in the solution must not exceed a prescribed upper limit. In many applications, vehicles must return to the depot within a time limit due to obvious operational reasons such as the duty hours of the driver. A distance limit may be applicable if the fuel tank of the vehicle has limited capacity and refueling facility is available only at the depot.

### 4.2. Vehicle routing with time windows

Many times the customers accept deliveries only during a specific time slot during the day, e.g. from 8 a.m. to 11 a.m. or 2 p.m.–5 p.m. etc. A typical example is the home delivery of groceries from online grocery stores, where the customer chooses a specific time slot, e.g. 2–4 p.m., during which he/she is available at home to receive the delivery. If the vehicle reaches the customer before the opening of the time window, it has to wait leading to wasted time. On the other hand, if it reaches after the closing time of the window, the delivery is not possible, and such a solution is clearly unacceptable. Sometime, the presence of tight time windows may make the vehicle capacity redundant and irrelevant. For example, even if the vehicles had a vary large capacity, a single vehicle may not be able to serve all customers, and multiple vehicles may be required in order to satisfy the time window constraints. When modeling this variant, the mathematical model must keep track of the time at which the vehicle reaches a particular customer so as to enforce the time window constraints. This requires introducing additional variables in the model, making the problem more difficult to solve. For a recent important work on this model, the reader is referred to Lysgaard (2006).

*4.3. Simultaneous pickups and deliveries*

Many times the product being delivered is stored in containers, crates or other packaging such as bottles. The empty crates or bottles from the previous day/week must be brought back to the depot from each customer. Typical example of this is the soft drinks or milk delivered in glass bottles stored in crates. The vehicle delivers filled crates to a customer, and picks up the empty crates from the previous day to bring them back to the depot or the bottling plant. If the quantity delivered is different from the quantity picked up at each customers, it gives rise to a new challenge. Even if the total delivery as well as the total pickup on the route is well within the vehicle capacity, the route may still not be feasible. For example, consider a route with three customers whose pickup quantities are (4, 3, 2) and delivery quantities are (2, 4, 3). If the vehicle capacity is 10, then both the total pickup as well as the total delivery satisfy the vehicle capacity. However, if the customers are visited in the order D- > 1- > 2- > 3- > D the route is not feasible. The vehicle starts with a total load of 9 from the depot. At node 1, It delivers 2 units and picks up 4 units, resulting in a total load of 11, which exceeds the vehicle capacity. This class of problems are particularly hard to solve due to the above consideration, although some recent papers (e.g. Agarwal & Venkateshan, (Agarwal & Venkateshan, 2020) and (Agarwal & Venkateshan, 2022)) have extended the current state of the art.

*4.4. Vehicle routing with common carrier*

In some situations it may not be the most economical option to make all deliveries using the dedicated private fleet of the shipper. For example, if a small quantity is to be delivered to a remote location, it may be quite expensive send a private vehicle to make such a delivery. Such a location may be served much more economically using the common carrier (also known as public carrier) service, which is somewhat akin to a courier delivery. The common carrier companies have fixed tariffs for making deliveries to the locations served by them, and the charges depend on the distance as well as the quantity to be delivered. In such a scenario, an additional input to the model is the cost of making each delivery by common carrier. The model then has to decide which locations should be delivered by common carrier and which by private truck so as to minimize the overall cost.

*4.5. Vehicle routing with profits*

In many situations, there is a revenue associated with making the delivery, and the transporter may choose to accept a subset of deliveries that maximize the profit, i.e. total revenue collected from the deliveries minus the cost of making those deliveries. The optimization model has to perform a trade-off between the cost of making the delivery and the revenue earned from it. This problem is particularly challenging as it involves a complex trade-off. If it was possible to work out a specific cost of delivery to each customer, then problem would be easy. By subtracting this cost from the revenue, we get the profit earned from each delivery. Then we can select the deliveries with maximum profit subject to the truck capacity constraint. However, It is not possible to pinpoint the delivery cost for a location, as it depends on which other customers are served by the same vehicle, and in what sequence. In order to model this problem, we need additional binary decision variables $y_i$, where $y_i = 1$ if the transporter chooses to make delivery to location $i$, and $y_i = 0$ otherwise. The objective function of the model is to maximize $\sum_i r_i y_i - \sum\sum c_{ij} x_{ij}$, i.e. the revenue earned from the chosen set of deliveries minus the cost of making those deliveries. Presence of additional binary variables in this model adds to the complexity of solving the problem.

*4.6. Vehicle routing with drones*

The use of unmanned aerial vehicles or drones, is being increasingly adopted to deliver small packages to customers. While the drones have some advantages, they also have some limitations. The main advantage or a drone is its low cost of operation compared to a vehicle. A drone can easily reach areas which may be difficult to reach by a vehicle due to traffic congestion etc. However, drones have a limited flying range due to limited battery capacity. The size/weight of shipments that can be delivered by drone is also limited due to the carrying capacity of the drone.

This gives rise to an interesting hybrid mode of operation, where a drone accompanies a delivery vehicle. The vehicle stops in the close proximity of a cluster of locations accessible by the drone, which makes multiple sorties to make these deliveries and comes back to the vehicle. The vehicle then moves on to another location, where this mode of operation is repeated. It is assumed that the drone batteries can be recharged on the vehicle.

The optimization model must decide as to which customers should be served the vehicle vs. the drone, and the stopping points of the vehicle from where the drone is launched to make those deliveries. This results in a challenging optimization problem which has only recently begun to receive attention of the research community. See Tamke & Buscher, (Tamke & Buscher, 2021) for a recent work.

## 5. Scope for future research

VRP is a very heavily researched problem. A Google Scholar search for "Vehicle Routing Problem" returns 6,35,000 entries. However, that does not mean that there is no scope for further research on this problem. Newer and newer variants of the problem keep getting identified posing fresh challenges. Being an NP-hard problem, it is quite difficult to solve the problems of even moderate sizes optimally. Therefore, a large body of literature is devoted to finding heuristic methods for solving the problem. Newer variants of the problem pose fresh challenges for finding effective and efficient heuristics for solving these variants. Therefore, the Vehicle Routing Problem and its variants remains a very fertile area for researchers.

## Conflicts of interest

There is no conflict of interest.

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