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Is CAPM Valid: Search for Efficient Market Portfolio

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Abstract

The selection of a proxy for market portfolio in CAPM model remains one of the most crucial steps in its empirical use. This paper aims to check the validity of CAPM in Indian market and to identify the most efficient proxy for market portfolio. Incorrect selection of this proxy may lead to inaccurate result and even rejection of CAPM model when tested empirically. This further emphasises the importance of selecting the correct proxy for market portfolio. The paper reviews why standard cap-weighted indices can be used as the proxy and then which of such index is the most efficient proxy for the Indian market. A sample of 100 companies and has been taken to study the efficiency of four broad index namely BSE SENSEX, BSE 100, BSE 200, BSE 500 as proxy for market portfolio. The result shows that BSE 500 is a better index to be used as proxy for the market portfolio, instead of the most popular BSE Sensex, which is widely used for these kinds of studies.

INTRODUCTION

Capital asset pricing model (CAPM) is a set of predictions concerning equilibrium expected return on risky asset. Harry Markowitz (1952) laid down the foundation of modern portfolio theory and CAPM was developed later in articles by William Sharpe (1964), John Linter (1965) and Jan Mossin (1966). The attraction of the CAPM is that it offers powerful and simple predictions about ways to measure the risk and also the association between expected return and risk. The foundation of CAPM was laid on some simplifying assumptions. For example, as per the CAPM the risk of a stock should be measured relative to a comprehensive "market portfolio".

The question that needs to be answered is "what is a market portfolio" ? When we sum over or aggregate, the portfolio of all individual

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investors, lending and borrowing will cancel out and is market portfolio is thus the value of the aggregate risky portfolio which is equal to the entire wealth of economy. As we primarily deal with securities so we can just include all traded financial securities and can easily omit consumer durables, real estate and human capital. The proportion of each stock in this portfolio is equal to market value of the stock divided by the sum of market value of all the stocks. The CAPM implies that as individuals attempt to optimize their personal portfolios, each of them arrive at same portfolio with weights on each asset equal to their weights in the market portfolio M. And for this reason, the risk of an individual security must be measured in term of the extent to which it adds risk to the investor's portfolio. Thus, a security's contribution to portfolio risk is different from the risk of the individual security. Not only will the market portfolio be on the efficient frontier, but will also be the tangency portfolio to the optimal capital allocation line (CAL) derived by each and every investor. As a result the capital market line (CML) is best attainable capital allocation line. All investors thus, hold M as their optimal risky portfolio, differing only in the amount invested in it versus the amount in the risk-free asset. The risk premium on individual assets will be proportional to the risk on the market portfolio, M, and the beta coefficient of the security relative to the market portfolio.

The capital asset pricing model (CAPM) says that the "market portfolio" is mean variance optimal. The ex-ante construction of a mean-variance efficient portfolio as suggested by Markowitz (1952) is a difficult problem. Forecasting expected stock returns and their covariance matrix of thousands of stocks - which is necessary for applying Markowitz's mean-variance portfolio construction (1952) is intellectually challenging and resource intensive. This is precisely why CAPM remains so powerful. If one can find the market portfolio, one simultaneously identifies a mean variance-optimal portfolio.

The investment industry and countless MBA programs have promoted the belief that cap weighted equity market indexes are sufficiently representative of the CAPM market portfolio to be nearly mean-variance efficient. When we accept this simplifying assumption, we reduce the complicated problem of optimal portfolio construction to essentially buying and holding a cap-weighted index. Further cap-weighted index is based on capitalization weighting, which is a passive strategy requiring little trading; thus reducing trading costs. The cap-weighted indices require material adjustment only when new companies become large enough to merit inclusion in an index or when others disappear through merger, failure, or relative changes in capitalization, collectively referred to as "reconstitution." Such changes are not insignificant. A study of changes in the composition of the S&P 500 (Blume and Edeien 2003) found that nearly half, 235 companies, had been replaced between 1995 and 2000. Capital weighting tends to emphasize on large companies and also the more heavily traded stocks, thereby further reducing portfolio transaction costs.

The most popular capital weighted index is BSE Sensex, which is a portfolio of 30 large and highly traded companies. The selection of these companies is done such that almost all sectors of economy are represented by this index. It is for these reasons it is also called as the barometer of Indian economy. But then the question arises whether popularity also indicates that it is the most efficient proxy for market return which can be correctly used in CAPM. This very question is the genesis of this paper.

The paper explores the possibilities of using other broad index as BSE 100, BSE 200, BSE 500 along with BSE Sensex, to be used as most fit proxy of market portfolio. The paper is divided into four sections. The first Section deals with the model of the study, Section 2 describes the database and methodology, Section 3 describes the empirical findings of the study and the implications thereof. The last section concludes the study with summary and conclusions.

THE MODEL OF STUDY

As discussed in the introduction, the main objective of this research paper is to identify which of the broad index available in India can be used as a best proxy for market portfolio. This Market proxy can then be used in determining the market risk premium. This section discusses the theoretical model that can be used to test the market risk premium using different broad index as proxy for market portfolio and then selecting the best among them.

The Capital Asset Pricing Model

The CAPM conveys the notion that securities are priced so that the expected returns will cover expected risks for the investors. There are two fundamental relationships: the capital market line and the security market line. These two models are the building blocks for deriving the CAPM. Even though they are not new, it is illustrative to discuss them here briefly. Further, the model used for the analysis is a derived form of CAPM, thus these models deserve some attention in this paper.

Capital Market Line

The capital market line (CML) specifies the return an individual investor expects to receive on a portfolio. This is a linear relationship between risk and return on efficient portfolios that can be written as:

$$E(R_p) = R_f + \sigma_p \left(\frac{E(R_m) - R_f}{\sigma_m} \right) \qquad (1.1)$$

where,

R_p = portfolio return,

R_f = risk-free asset return,

R_f = market portfolio return,

σ_p = standard deviation of portfolio returns, and

σ_m = standard deviation of market portfolio returns.

According to the equation, the expected return on a portfolio can be thought of as a premium for bearing risk inherent in the portfolio. The CML is valid only for efficient portfolios and expresses investors' behavior regarding the market portfolio and their own investment portfolios.

Security Market Line

The security market line (SML) expresses the return an individual investor can expect in terms of a risk-free rate and the relative risk of a security or portfolio. The SML with respect to security i can be written as:

$$E(R_i) = R_f + \beta_p \left(E(R_m) - R_f \right) \qquad (1.2)$$

Where,

R_i = security return

R_f = risk-free asset return,

R_m = market portfolio return

β_i = Beta which is amount of non-diversifiable risk inherent in the security relative to the risk of the market portfolio.

Single-factor CAPM

In order to test the validity of the CAPM researchers always test the SML given in (1.2). The CAPM is a single-period ex ante model. However, since the ex ante returns are unobservable, researchers rely on realised returns. The beta in such an investigation is usually obtained by estimating the security characteristic line (SCL) that relates the excess return on security i to the excess return on some efficient market index at time t. The ex post SCL can be written as:

$$r_{it} - r_{ft} = \alpha + \beta_i (r_{mt} - r_{ft}) + e_{it} \qquad (1.3)$$

Where,

$\overline{r_{it} - r_{ft}}$ = Sample average of the excess return on each of the 100 stock.

β_i, = Sample estimates of beta coefficient of each of 100 stocks.

$\overline{r_{mt} - r_{ft}}$ = Sample average of excess return of the market index

$\sigma 2\,(e_i)$ = Estimates of variance of the residual for each of the 100 stocks.

The estimated β i is then used as the explanatory variable in the following cross-sectional equation:

$$\overline{r_i - r_t} = \gamma + \gamma_0 + \gamma_1 \beta_i \qquad (1.4)$$

The coefficient γ_0 is the expected return of a zero beta portfolio, expected to be the same as the risk-free rate and γ_l is the market price of risk (market risk premium), which is significantly different from zero and positive in order to support the validity of the CAPM.

When testing the CAPM using (1.3) and (1.4), we are actually testing the following issues: (i) bi s are true estimates of historical âi, (ii) the market portfolio used in empirical studies is the appropriate proxy for the efficient market portfolio for measuring historical risk premium and (iii) the CAPM specification is correct (Radcliffe, 1987).

For each stock, i, we estimate the beta coefficient as the slope of a first pass regression. (The terminology first pass regression is because of the fact that the estimated coefficients will be used as inputs into the second pass regression.) Comparing equation (1.2) and (1.4) it can be concluded that if Rm has come from a correct proxy for market the CAPM is valid, then γ_0 and γ_l satisfy

$$\gamma_0 = \text{o and } \gamma_1 \overline{R\, m - Rf}$$

The bar indicates the average market risk premium. The hypothesis that γ_0 is consistent with the notion that nonsystematic risk should not be priced, that is, that there is no risk premium earned for bearing nonsystematic risk. Also according to CAPM, the risk premium depends only on beta. Therefore any additional right-hand side variable in equation 1.4 beyond beta should have a coefficient that is significantly different from zero in the second pass regression. Further, since firm is very small as compared to the market the variance due to the residual term u will be very insignificant and can safely be taken as zero.

Thus if we say that securities beta from the first pass regression are true estimated and can be used as inputs to the second pass regression, the CAPM will hold if and only if the market index used in the analysis is surely the " market portfolio" of the CAPM.

DATABASE AND METHODOLOGY

The empirical investigation of the present study is based on returns of 100 firms belonging to the BSE 100, for the period of 1999 - 2005. This gives data for 7 years thereby giving 84 monthly holding periods. For each of these monthly holding period, returns on these 100 stocks has been collected. The basic source of the data is Prowess electronic database of CMIE. Implied one month T-bills rate were taken from the database of Reserve bank of India and these rates are used as the risk free rate. Return on four index i.e. BSE Sensex, BSE 100, BSE 200, and BSE 500 were also collected from Prowess. These index returns has been taken one by one as proxy for the market portfolio. Thus the data consist of

R *it* Return on the 100 stocks over the 84- month sample period: i = 1, ...100, and t = 1, ... 84

R *ft* Risk free rate each month

R *mt* Return on market index over the sample period where analysis has been performed for each of the four market index taking one at a time

For each stock, i, we estimate the beta coefficient as the slope of a first pass regression equation

$$R_{it} - R_{ft} = \eta_i + b_i (R_{mt} - R_{ft}) + \varepsilon_{it} \qquad (1.3)$$

where, η_i is the constant return earned in each period and β_i is an estimate of bi in the SML. Based on this following statistics is used in later analysis

$\overline{R_{it} - R_{ft}}$ Sample averages (over the 84 observation) of the excess return on each of the 100 stocks.

bi Sample estimates of beta coefficients of each of the 100 stocks.

$\overline{R_{it} - R_{ft}}$ Sample average of the excess return of the market index.

$\sigma^2(ei)$ Estimates of the variance of the residuals for each of the 100 stocks.

The sample average excess return on each stock and the market portfolio are taken as estimates of expected excess return, and the value of bi are estimates of the true beta coefficient for the 100 stocks during the sample period. The $\sigma^2(ei)$ estimates the non systematic risk of each of the 100 stocks.

Now using the characterstics of security market line and 100 observation for the stock in our sample we can estimate ã0 and ã1 in the following second-pass regression equation with the estimates of bi from the first pass regression as independent variable.

$$\overline{Ri - Rf} = \gamma_0 + \gamma_1 bi \qquad i = 1, 2, \ldots\ldots.100 \qquad (2.1)$$

Comparing equation (1.2) and (1.4) it can be concluded that if R_m has come from a correct proxy for market the CAPM is valid, then γ_0 and γ_1 satisfy two hypothesis

$$\gamma_0 = o \text{ and } \gamma_1 = \overline{Rm - Rf,}$$

Proper selection of market proxy can thus be validated if ã1 comes out to be statistically significant with $\gamma_1 = Rm - Rf$ and γ_0 comes out to be statistically insignificant. This process has been repeated with all four market index as proxy for the market portfolio to determine the best proxy.

EMPIRICAL RESULTS

As discussed in the methodology the best proxy for the market portfolio can be identified using the second pass regression equation 2.1. Each of the four broad market indexes has been used one by one to estimate the beta coefficient for each of the 100 stocks over the 84 months sample period. These estimates were then used in the second pass regression equation which gives different models as per equation 2.1 for different market index. The results are as shown in the table

When market proxy is ↓	For γ_0		For γ_1		$\overline{Rm - Rf}$
	Value	Significance	Value	Significance	
Sensex	37.1637	6.26E-06	1.3405	0.853877	5.71354
BSE 100	29.77587	8.5E-05	10.33204	0.200228	8.723403
BSE 200	26.0901	0.001031	13.96343	0.090333	10.14861
BSE 500	24.77143	0.001668	15.59891	0.059683	11.56888

Here the point to be noted is that the coefficient values for ã1 in fact are the coefficient bi which should be equal to R m - R f, the market risk premium.

Starting with the most popular index, that is BSE Sensex, the second pass regression results shows that the coefficient of ã0 is not only greater than zero but it is also highly significant even at 1% level of significance. And the coefficient of bi is neither coming close to R m - R f nor the value is statistically significant. Thus both of our hypotheses are not supported when we take the BSE Sensex as a proxy for the market portfolio. One of the reasons for this behavior may be the small number of stocks in BSE Sensex. As there are only 30 stocks this index may fall short in completely explaining the variance of a relatively huge market portfolio.

When the proxy for market portfolio is taken as BSE 100 the results are somewhat same as that of BSE Sensex. Here also the coefficient of ?0 is greater than zero and also it is highly significant even at 1% level of significance. And coefficient of bi is also neither coming close to R m - R f nor is the value statistically significant. Again it can be inferred that BSE 100 index is also not a good proxy for the market portfolio.

The regression result with BSE 200 as the proxy for the market portfolio indicates coefficient of ?0 is greater than zero as well as statistically significant. Here, although the coefficient of bi is coming out to be significant at 1 % level of significance, it is very clear that its value is not close to R m - R f. Thus BSE 200 offers better proxy for market portfolio than BSE Sensex and BSE 100 in the scene that its value is at least statistically coming significant but again it is not a very efficient proxy for the market.

But the results with BSE 500 as the proxy for the market are quite interesting. Here the value of ã0 is greater than zero and it is significant at 1 % level of significance. Also the coefficient of bi, that is, the value of γ_1 is coming out to be more significant compared to all other proxies of market taken above. The value for this is 15.59891 which is also close to 11.56888 which is the value of R m - R f. Though these values are not exactly equal to our hypothesized values yet they are very close to them, and when compared to the values with respect to the values of other market proxies the results are relatively convincing, and thus it is suggested to use BSE 500 as the proxy for the market portfolio. This finding is also in line with the fact that BSE 500 is a bigger index in terms of the number of stocks included in it and hence is able to capture the overall market portfolio better than other indices.

Also, it can be noted that the value of ã0 is coming out to be positive for all the market proxies, this indicates the undervalued state of the market indices.

Thus, the use of each of the four market index as proxy for the market portfolio in the single index model, one by one, reveals that using BSE 500 as a market proxy is far better that using any other index including the most popular index BSE Sensex.

SUMMARY AND CONCLUSION

The capital asset pricing model (CAPM) says that the "market portfolio" is mean variance optimal. There are some simplifying assumptions which should be satisfied before CAPM can be used for empirical purposes. One of the assumptions lies in determining the proxy for the market portfolio. Single index model- which is a form of CAPM- can be used to test the efficiency of various indices as market proxy and to judge the usefulness in the use of CAPM.

Capital weighted indices are most popular, as a proxy for market portfolio. In India historically BSE Sensex is the most popular index. But the test of market risk premium indicates that BSE 500 is a better proxy for the market portfolio not only against the BSE Sensex but also against the other two broad indexes BSE 100 and BSE 200. Further no matter which proxy is used, the study indicates that markets are undervalued.

The result of this study is that, the BSE 500 index should be used as proxy for market portfolio in CAPM, where ever CAPM model is empirically applied. This will help in formulating better results and taking better decisions, using the most accepted pricing model that is the CAPM model.

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