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EFFICIENCY OF BLACK AND SCHOLES MODEL FOR PRICING OPTIONS AT INDIAN STOCK MARKET

Vaibhav Kaushik*

ABSTRACT

Black and Scholes model was developed for valuing the options in derivatives market in the year 1973. This model became the prime tool for pricing options but because of its deficiencies it forced researchers to find out new model for pricing options (Rubinstein, 1985; Hull and White, 1987; Wiggins, 1987; Dumas, Fleming and Whaley, 1998). Option pricing is proven to be very challenging task during high volatility situations whereas Black and Scholes option pricing model is not proven a successful predictor for determining approximate price of the options. Even empirical evidences support that Black and Scholes model produces some biasness in estimating the prices. The main objective of this paper is to find out the efficiency of the Black and Scholes option pricing model for predicting the option prices at Indian Stock Exchanges for which data set from Indian Stock Exchanges will be chosen, market price of option contracts are considered as sample for the year 2008-2015. The actual market price will be compared with the price derived with the help of Black and Scholes model formula.

Keywords: *Options, Black and Scholes model, Derivatives, Volatilities, Indexation*

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BACKDROP

Black and Scholes model is developed in 1973 under the concepts of financial derivatives for valuing the options in derivatives market; financial derivatives are more dependent on the usage of financial mathematics. Over the years few attempts were made to find out the efficiency of Black and Scholes option pricing model for valuing options contracts. Generally the empirical evidences on the same studies found that Black and Scholes model misprices the options to the extent and volatilities are high for in the money options especially. The major problems which are found in Black and Scholes formula for these volatilities are due to its dependence on few assumptions, for deriving the theory concept is depended on few assumptions which may or may not necessary be true all the times. Option pricing is proven to be very challenging task during high volatility situations where black and Scholes option pricing model is not proven a successful predictor for determining approximate price of the options. Even empirical evidences support that black and Scholes model produces some biasness in estimating the prices. Keeping all these factors in mind this paper evaluates the extent of appropriateness of Black and Scholes models for determining option prices at Indian stock exchanges. This paper checks out how Black and Scholes model works for predicting prices at Indian stock exchanges.

LITERATURE REVIEW

The developer of Black and Scholes model themselves tested the efficiency of the model in the year 1972 and found that the result of formula gives lower value than the actual market data.

1973- 1983

Merton (1973) extended the Black and Scholes formula and showed that the basic form of model was the same if the payment structure was increase or lifted, the interest rate is stochastic and the option is exercisable prior to maturity. **Thorp** (1973) found that Black and Scholes formula holds true and gives the nearest value to actual market value if the regulators keep some restrictions on proceeds of short sales. **Merton, Cox and Ross** (1975) have found that if the return on ordinary stock do not follow a stochastic process with a continuous path, the hedge mechanism used by Black and Scholes will not be suitable.

Latane and Rendleman (1976), **Mac-Beth and Merville** (1979) solved the Black and Scholes formula in the form of implied variance rates by taking a sample of period 1975-1976 and found a result that strike price is biased but this bias was exactly opposite which is reported by Black and Scholes.

Galai (1977) used the data of Chicago Board Option Exchange and found that additional daily returns on hedged portfolio are varying from zero and transaction cost eliminates the excessive positive return. **Bhattacharaya** (1980) tested Black and Scholes model and found the over valuation of model for at the money options, opposite to that near the money options were undervalued. **Geske, Roll and Shastri** (1983) found that the reason behind the formula value and actual market price is due to dividend protection in the OTC market.

2000- 2016

Ramazan Gencay and Aslihan Salih (2003) found that Black-Scholes mispricing worsens with increasing volatility and feed forward networks handle pricing during high volatility with considerably lower errors for out-of-the-money call and put options. Orhun Hakan Yalincak (2005) found that Black and Scholes model doesn't consider few relevant aspects of the market to predict the option price. S. McKenzie, D. Gerace and Z. Subedar (2007) tried to find out the effectiveness of the Black and Scholes formula on Australian stock exchange and found the model relatively accurate in pricing options.

Khan et al., (2012) suggested few modifications in Black and Scholes model on the grounds of risk free rate assumptions. **Sarbapriya Ray** (2012) found that Black and Scholes model is lacking in few assumptions and more relevant and practical assumptions should be made to derive proper option price.

Matloob Ullah Khan, Ambrish Gupta and Sadaf Siraj (2013) also recommended some modification on the assumption of risk free interest rate in Black and Scholes formula. **Rachna Aggarwal and Rajesh Kumar** (2013) found that out of the options are under-priced by this model in the Indian option market. They found that money options are generally overpriced by this model. **Vipul Kumar Singh** (2013) compared the results of Black and Scholes performance with the other models for deriving option prices and found that Black and Scholes model is the model producing the highest pricing errors.

Jaakko Salminen (2015) attempted to compare Black and Scholes model with two other advanced models for valuing options and found that other models were quite efficient in predicting the option price compared to Black and Scholes formula.

Matthew J. Krznaric (2016) found that Black and Scholes model is not too accurate model to predict the actual option prices.

Data Analysis

For each data set approximated underlying stock price and option price are calculated according to average of last bid and ask quotes. The risk free rate is collected from 91 day government's treasury bills rate mentioned during the issue date of the option.

Researchers acknowledge biasness unavoidable in volatility estimation. For ensuring robustness in results three different measures of volatility are used: historical instantaneous (v1) actual instantaneous (v2) and implied volatility (v3). The instantaneous measures are derived by standard deviation of the underlying stock returns.

The implied volatility is the value of σ that when substituted into Black Scholes model equates the price of the option to the observed market price.

$$C = S0\,N(d_1) - Ke^{-RFT} N(d2) \tag{1}$$

$$d_1 = \frac{\ln(S0 / K) + (RF + \sigma^2 / 2)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = \frac{\ln(S0) / K) + (RF - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{3}$$

It is impossible to invert the Black-Scholes equation so that σ is expressed as a function of S0, K, T, RF and C. A root finding technique is implemented to calculate implied volatility.

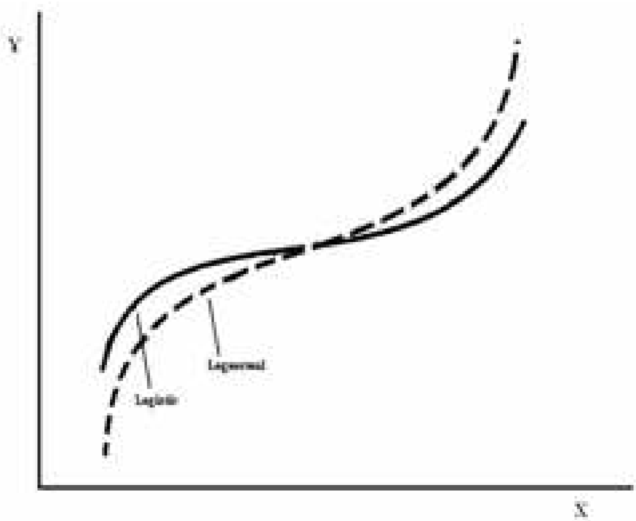
Model

Research paper focuses on log distribution that will increase the tail properties of lognormal distribution. The same distribution was used in work by Draper and Smith (1981) and Aparico and Estrada (2001).

Logistic and lognormal distributions are same but their base for different significance level differs. Logistic distribution's fatter tails suggest that conditional probability approaches 0 and 1 at slower rate compared to lognormal distribution.

In logistic distribution predicted probability that option will be exercised at maturity is lower compared to lognormal distribution at levels more than 50% and lowers at levels less than 50%.

Figure-1 Logistic and lognormal underlying distributions



The investigation of this paper is expressed in binary form where Y is Bernoulli Variable. If the option gets exercised at maturity, Y is 1 or not exercised Y is 0. This paper uses regression models logit and probit to test the mathematical significance of Black and Scholes model with Indian stock observations.

Logit model formula on the basis of logistic distribution is

$$P_i = (X=1 \mid SO, K, T, RF, V_i) = F(\beta_0 + \beta_1 SO + \beta_2 K + \beta_3 T, \beta_4 RF + \beta_5 V_i) \quad (4)$$

$$= \frac{1}{1 + e^{-(\beta_0 + \beta_1 SO + \beta_2 K + \beta_3 T, \beta_4 RF + \beta_5 V_i)}}$$

X= 1 if the option is exercised at maturity and 0 if it is not;

Pi is the probability that option will be exercised at the date of maturity. X is binary dependent variable; φ is cumulative lognormal distribution function; S0 is price of stock; K being strike price; T means time to maturity; RF is risk free rate; VI is Volatility (V1, V2 and V3); Bi is coefficient regressor.

The dual stage least square regression includes two consecutive regression applications. The first step regression model is qualitative regression model.

The logit model estimation;

$$P = (X = 1 \mid So, K, T, Rf, V_i) = F(\beta_0 + \beta_1 So + \beta_2 K + \beta_3 T, \beta_4 Rf + \beta_5 V_i) \quad (5)$$

The probit model estimation;

$$P_i = (X = 1 \mid So, K, T, Rf, V_i) = \varphi(\beta_0 + \beta_1 So + \beta_2 K + \beta_3 T + \beta_4 Rf + \beta_5 V_i) \quad (6)$$

The models discussed above distributes Pi into two components; problematic component that can be attributed with the error term and another problem free component that is not attributed with the error term. The second step includes the problem free component to predict the level of β1. The level of β1 checks the statistical significance of individual regression model.

Interpretation

Logit and Probit models show X=1 if call option was exercised and X=0 if not. The independent variables used in model are factors used in Black and Scholes option pricing formula , price of stock (S0), Option' s strike price (K), Time to maturity (t), Risk free rate (RF), Historical volatility (V1), Actual volatility (v2), and Implied volatility (v3). The results are given in tabular form in table -1 and table -2. Maximum likelihood method is used for logit model, so standard errors are asymptotic. Z statistics is used to test the level of significance for coefficient.

The predicted model is highly significant at 1% level by associated p values. The McFadden R 2 ranges among 0.164 (column 1 and 2) to 0.630 (column 3) highlighting that 62-63% options exercised on Indian stock exchanges were predicted accurately by logit model.

Each slope of coefficient in the model is partial slope coefficient and mentions change in predicted logit model for a single unit of change in value of given regressor. Coefficient in first regression assumes in table 5-1 of 0.745 suggesting that if S0 increases by a single unit, logit model increases by 7.475 units, indicating positive relationship between two keeping other variables constant.

Table-1 **Logit Model Pi** $= (X = 1\ SO, K, T, RF, V) = F(\beta_0 + \beta_1 SO + \beta_2 K + \beta_3 T, \beta_4 RF + \beta_5 V_i)$

Columns are three regression estimates. Dependent variable option payoff at the end of the contract: 1 if exercised 0 if not. Mean standard errors of individual coefficient are in brackets. S0 price of stock at beginning ; K Strike price; T time to maturity presented as percentage of number of Indian stock exchange's trading days; RF risk free rate of interest ; V1 volatility of stock before life of option; V2 actual volatility of stock over life of option ; V3 implied volatility ; LR is ratio of degrees of freedom at 5.

Explanatory Variables	(1)	(2)	(3)
Intercept	-5.6134* (3.7479)	-5.8813* (3.7809)	-5.4564* (3.7738)
SO	0.7475*** (0.2101)	0.7553*** (0.2109)	0.7454*** (0.2069)
K	-0.7048*** (0.2008)	-0.7104*** (0.2015)	-0.7029*** (0.1977)
T	1.9455*** (0.5718)	1.9737*** (0.5750)	1.9070*** (0.5766)
RF	107.1126** (66.9243)	109.0983** (68.2495)	105.7904** (68.0872)
V1	0.0484 (0.3866)	- -	- -
V2	- -	0.3538 (0.6661)	- -
V3	- -	- -	0.4099 (1.0226)
LR statistic (5 df)	35.477	35.646	35.706
p-value	0.000	0.000	0.000
McFadden R^2	0.164	0.165	0.185
Count R^2	0.623	0.623	0.630
Sample size	129	129	129

Table 5-1 suggests that all the other regressors apart from option strike price (K) have positive impact on logit model, suggesting that all variables are significant. The intercept and risk free interest rate are significant at 10% level, all other variables excluding volatility measures are significant at 1% level. Every volatility measure (V1,V2 and V3) are insignificant indicating that volatility does not have effect on probability of a European call option being exercised on Indian stock exchanges. However combined all regressors have significant impact on predicted probability; LR statistic of individual equation is among 35.477 and 35.646 and p-values < 0.0001.

Each of the slopes co-efficient in probit is a partial slope coefficient and tests change in predicted probit model for a single change in value of regressor. So coefficient in first regression estimate in table 5-2 of 0.4191 means, if S0 rises by one unit average probit model rises by 4.191 units, indicating positive relationship between both of them. Probit and logit model are same but their predicted co-efficient are not comparable.

Table 2 **Probit Model** $P_i = (Y = 1\ SO, K, T, RF, V_i) = \varphi(\beta_0 + \beta_1 SO + \beta_2 K + \beta_3 T, \beta_4 RF + \beta_5 V_i)$

Explanatory Variables	**(1)**	**(2)**	**(3)**
Intercept	-4.1220* (2.2540)	-4.2047* (2.2682)	-4.0581* (2.2739)
S0	0.4121*** (0.1101)	0.4213*** (0.1101)	0.4203*** (0.1067)
K	-0.3964*** (0.1064)	-0.3975*** (0.1067)	-0.3976*** (0.1050)
T	1.3071*** (0.3373)	1.2641*** (0.3390)	1.1838*** (0.3497)
RF	67.9896** (39.6969)	68.8859** (39.8319)	66.9653** (39.8645)
V1	0.0418 (0.2425)	- -	- -
V2	- -	0.2037 (0.4174)	- -
V3	- -	- -	0.2673 (0.5748)
LR statistic (5 df)	34.303	34.435	34.600
p-value	0.00	0.00	0.00
McFadden R^2	0.163	0.164	0.165
Count R^2	0.702	0.707	0.713
Sample size	129	129	129

Predicted model here too is highly significant at 1% level using associated p-values. The McFadden R2 ranges among 0.163 and 0.165 mentioned in column 1 and 3 respectively suggesting that 61-63% options exercised on Indian Stock exchanges were predicted correctly by probit model.

Table 5-2 highlights positive impact of regressors on probit model excluding strike price (K), indicating economic significance of coefficients, intercept and risk free rate are significant at 10% level and other variables are significant at 1% level. Each volatility measure is not significant. Whereas jointly all the variables have a significant impact on the model.

Observing between the logit and probit variables, analysis of data for each equation in table 5-5 and 5-6 is necessary. LR data for each model shows that logistic model displays higher LR numbers compared to probit model equation. The same results were found under literature of Duan (1999) that Black and Scholes model produces accurate results when tail properties of distribution are increased.

Table-3 Second Stage Least Squares $R_{i,t} = \beta_0 + \beta_1 \hat{P_i} + \beta_2 R_{i,t-1}$

	V1			V2	V3
	φ	F	φ	F	φ
Intercept	-0.0211***	-0.0205***	-0.0198***	-0.0194***	-0.0205***
	(0.0099)	(0.0297)	(0.0099)	(0.0100)	(0.0298)
P	0.0315***	0.0287***	0.0286***	0.0283***	0.0289***
	(0.0122)	(0.0122)	(0.0122)	(0.0122)	(0.0122)
R2	-0.0041	-0.0041	-0.004	-0.004	-0.0041
	(0.0062)	(0.0062)	(0.0062)	(0.0062)	(0.0062)
Obs.	129	129	129	129	129

The level of B1 checks significance of each regression model. Analysis of significance for B1 for every regression model is at 1% level. Economical importance of B1 is highlighted with the sign of coefficient that is positive suggesting direct relationship between return of underlying stock and expected value of Pi.

Conclusion

After analyzing of the data in the paper, B and S model is accurately measuring the efficiency of the stock market. While comparison of regression models gives conclusion that Black and Scholes model is significant at 1% level in prediction of probability of option getting exercised. All the variables we are testing in regression model are significant. Whereas individually volatility variable is relatively less significant, suggesting volatility measures are irrelevant in predicting probability of option being exercised.

Qualitative regression models indicate significance of Black and Scholes model with logistic distribution over lognormal distribution. Using Jump diffusion anomaly raises the tail property of lognormal distribution, consequently rising statistical significance of Black and Scholes model. The scope of future research on Black and Scholes model is open for research scholars on its deficiencies and assumptions which forced them to find out new model for pricing options.

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